USN

Fifth Semester B.E. Degree Examination, December 2010
Modern Control Theory

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Consider a two input - two output system:

$$\frac{d^2y_1}{dt^2}+2\frac{dy_1}{dt}+3\frac{dy_2}{dt}+y_1=u_1+u_2 \quad \text{and} \quad \frac{d^2y_2}{dt^2}+2\frac{dy_2}{dt}+5\frac{dy_1}{dt}+y_2=2u_1+3u_2$$

Derive a state model of the system. **(06 Marks)**

b. Obtain the state model of the given network shown in Fig.Q1(b), in the standard form. $R_1 = 1\Omega$, $C_1 = 1f$, $R_2 = 2\Omega$, $C_2 = 1f$, $R_3 = 3\Omega$. Choose voltage across capacitor C_1 as e_1 and voltage across capacitor C_2 as e_2 as state variables. **(08 Marks)**

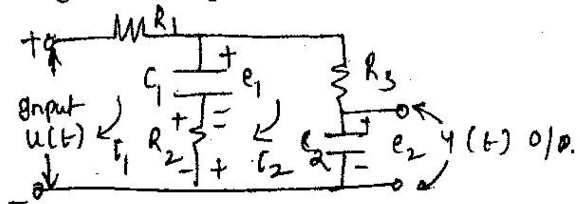

Fig.Q1(b)

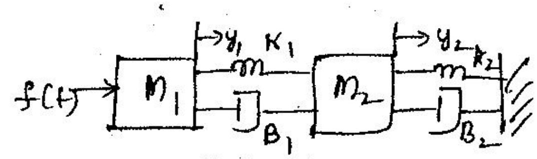

Fig.Q1(c)

c. Construct the state model of the mechanical system shown in Fig.Q1(c). **(06 Marks)**

2 a. A feedback system has a closed-loop transfer function $\frac{Y(s)}{U(s)}=\frac{10(s+4)}{s(s+1)(s+3)}$. Construct three state models and draw the block diagrams for each model. **(12 Marks)**

b. Reduce the given block model into its canonical form by diagonalizing the matrix A.

$$\dot{x}(t)=\begin{bmatrix}0 & 1 & -1\\ -6 & -11 & 6\\ -6 & -11 & 5\end{bmatrix}x(t)+\begin{bmatrix}0\\0\\1\end{bmatrix}u(t) \quad \text{and } y(t)=[1\ \ 0\ \ 0]\,x(t).$$ **(08 Marks)**

3 a. Obtain the transfer function of the system given as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -7 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}u \quad \text{and } y=[1\ \ 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$ **(06 Marks)**

b. For the speed control system, the following is the plant model:

$$\dot{x}=Ax+bu \quad \text{and} \quad y=cx$$

with $A=\begin{bmatrix}-1 & 1\\ -1 & -10\end{bmatrix}$; $b=\begin{bmatrix}0\\10\end{bmatrix}$; $c=[1\ \ 0]$

where, x_1 is the angular velocity of the shaft $w(t)$, x_2 is the armature current in $i_a(t)$ and $y(t)=x_1(t)=w(t)$. Obtain the response of the system, with zero initial conditions, for unit step input. **(14 Marks)**

4 a. Explain the concept of controllability and observability, with the conditions for complete controllability and observability in the S-plane. **(08 Marks)**

b. A state space representation of a system in the controllable canonical form is given by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -0.4 & -1.3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u \longrightarrow (1)$$

$$y=\begin{bmatrix}0.8 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \longrightarrow (2)$$

The same system may be represented by the following state space equation, which is in the observable canonical form

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & -0.4\\ 1 & -1.3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0.8\\ 1\end{bmatrix}u \longrightarrow (3)$$

$$y=\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \longrightarrow (4)$$

Show that the space representation given by equation (1) and (2), gives a system, that is state controllable, but not observable. Show, on the other hand, that the space representation defined by equations (3) and (4), gives a system that is not completely state controllable, but is observable. What causes the apparent difference in the controllability and observability of the same system? Explain. **(12 Marks)**

PART – B

5 a. Consider a linear system described by the transfer function $\frac{Y(s)}{U(s)}=\frac{10}{s(s+1)(s+2)}$. Design a feedback controller, with a state feedback, so that, the closed loop poles are placed at $-2, -1\pm j1$. **(12 Marks)**

b. Consider the system described by the state model $\dot{x} = Ax$ and $y = cx$, where $A=\begin{bmatrix}-1 & 1\\ 1 & -2\end{bmatrix}$; $c = [\,1 \quad 0\,]$. Design a full order state observer. The desired eigen values for the observer matrix are $\mu_1 = -5$ and $\mu_2 = -5$. **(08 Marks)**

6 a. Explain the properties of the nonlinear systems. **(08 Marks)**

b. With neat sketches, explain i) ON-OFF relay with deadzone ii) backlash iii) Saturation. **(06 Marks)**

c. What are the singular points? Explain the types of singular points. **(06 Marks)**

7 a. Explain the delta method of obtaining the phase trajectories. **(08 Marks)**

b. A linear second order servo is described by the equation $\ddot{e}+2\delta w_n\dot{e}+w_n^2 e=0$, where $\delta = 0.15$, $w_n = 1$ rad/sec., $e(0) = 1.5$, $\dot{e}(0) = 0$. Determine the singular point. Construct a phase trajectory, using the method of isoclines. **(12 Marks)**

8 a. Prove that $A^Tp + pA = -Q$ for linear time invariant systems. **(04 Marks)**

b. Explain the concept of i) Stability in the sense of Liapunov, ii) Asymptotic stability and iii) Instability. **(06 Marks)**

c. For the nonlinear system shown in Fig.Q8(c), obtain the stability using the Krasoviski's method, where nonlinear element is described as $u = g(e) = e^3$. The system is described by differential equation $\ddot{e}+\dot{e}=-Ke^3; K > 0$. **(10 Marks)**

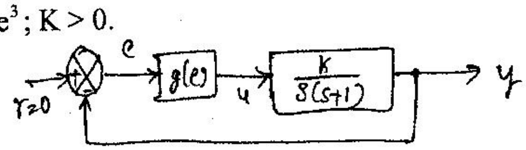

Fig.Q8(c)

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